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90635

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

Level 3 Calculus, 2005

90635 Differentiate and use derivatives to solve problems

Credits: Six
9.30 am Wednesday 16 November 2005

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Make sure you have a copy of the Formulae and Tables booklet L3-CALCF.

You should answer ALL the questions in this booklet.

Show ALL working for ALL questions.

Show any derivatives that you need to find when solving the problems.

If you need more space for any answer, use the page(s) provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–12 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

For Assessor's use only	**Achievement Criteria**	
Achievement	**Achievement with Merit**	**Achievement with Excellence**
Differentiate functions and use differentiation to solve problems. ☐	Demonstrate knowledge of concepts and techniques of differentiation. ☐	Solve problem(s) involving a combination of differentiation techniques. ☐
	Solve differentiation problems. ☐	
Overall Level of Performance (all criteria within a column are met) ☐		

You are advised to spend 50 minutes answering the questions in this booklet.

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Show **ALL** working.

QUESTION ONE

Differentiate the following functions.

You do not need to simplify your answers.

(a) $y = \sqrt{x} + 5e^{7x}$

(b) $y = \ln(\sin x)$

QUESTION TWO

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Find the equation of the tangent to the curve $y = \cos x$ at the point where $x = \frac{\pi}{6}$.

QUESTION THREE

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John's new company is producing video games and can sell all that it produces.

The revenue is given by: $R = 42x - \frac{x^2}{18}$,

where x is the number of video games produced per day and R is the revenue in dollars.

The number of video games produced per day is increasing at 20 games per day.

Find the rate of increase in revenue, $\frac{dR}{dt}$, when the production is 300 games per day.

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QUESTION FOUR

Find $\frac{dy}{dx}$ when $y = \frac{2x^2 + 1}{e^{3x}}$.

You do not need to simplify your answer.

QUESTION FIVE

Find $\frac{dy}{dx}$ if $x = 5\sec t$ and $y = 7\tan t$.

Simplify your answer.

QUESTION SIX

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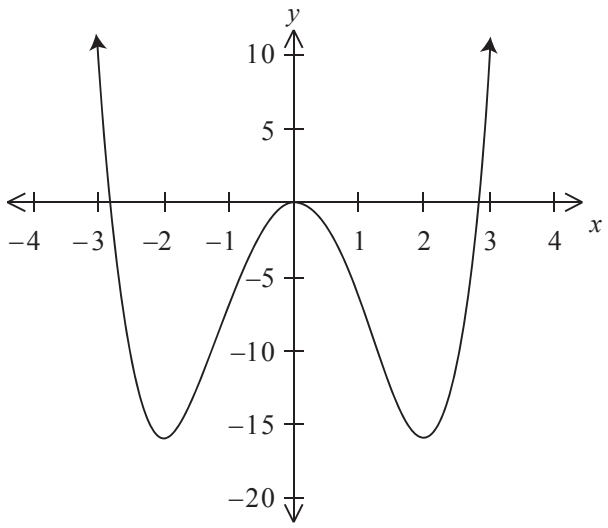
Here is a sketch of the graph of $y = x^4 - 8x^2$.



Find the values of x for which $y = x^4 - 8x^2$ is concave down.

QUESTION SEVEN

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Andrea is doing a project on the sale of T-shirts at concerts.

She finds that the profit depends on the number sold, given by the equation:

$$P = 25x - 20x \ln x$$

where x is the number of T-shirts sold in thousands and P is the profit in dollars.

Find the number of T-shirts needing to be sold to maximise the profit.

You may assume that $P''(x) < 0$.

You must show any derivatives that are needed to solve the problem.

QUESTION EIGHT

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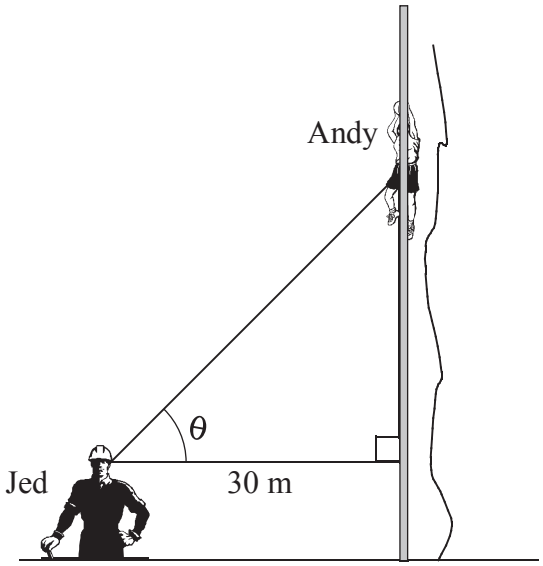
Jed watches Andy as he abseils down the cliff.
Jed is standing 30 metres from the base of the vertical rope.
Andy is descending the rope, down the cliff face, at a constant rate of 3 metres per second.
θ is the angle of elevation from Jed's eyes to Andy's position on the rope.

Find the rate of change of θ when Andy is 30 metres above Jed's eye level.



QUESTION NINE

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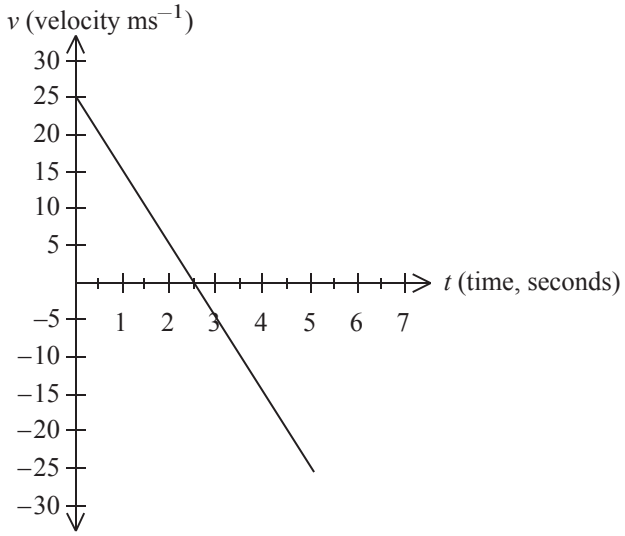
Angela and Jess are playing with a ball.

Angela kicks the ball vertically into the air.

The graph below shows the velocity function of the ball as it travels through the air for 5 seconds.



(a) When is the ball at its maximum height?

(b) What is the acceleration of the ball?

QUESTION TEN

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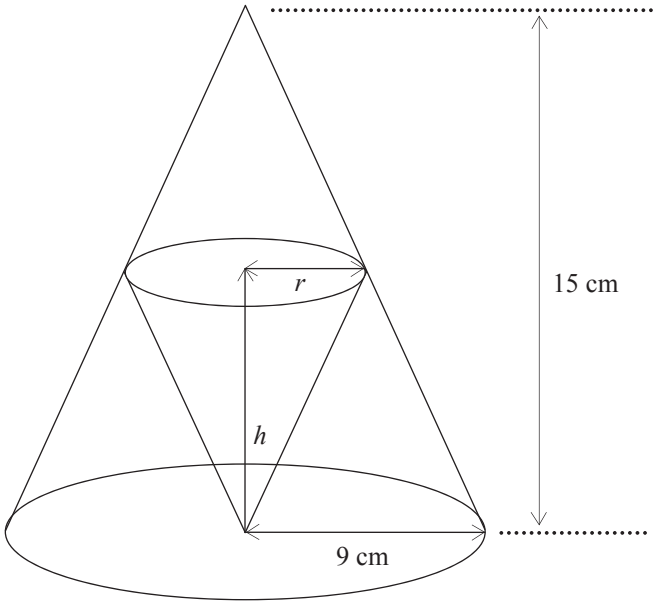
A cone has a height of 15 cm and radius of base of 9 cm.
Another smaller cone is inscribed in the cone.
Its vertex is at the centre of the base of the larger cone.
The base of the smaller cone is parallel to the base of the larger cone.

Find h and r, the dimensions of the smaller cone, so that it has maximum volume.
State the maximum volume of the smaller cone.



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Extra paper for continuation of answers if required.
Clearly number the question.

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Question number